3. The Trace Formula of Problem (1). Let us rewrite Eq. (11) in the form

$$\sum_{n=0}^{\infty} [\lambda_{mn} - (2m + 4n + 2) - A_m/\sqrt{n+1} - c_0 A_m / c\,(n + 1)^{3/2}] = 0, \tag{12}$$

where $c = \sum_{n=0}^{\infty} 1/(n + 1)^{3/2}$. Multiplying (12) by ε_m and summing over m, we arrive at the theorem.

THEOREM. Let q(r) be a continuous real-valued function equal to zero outside the interval [ε, a] (0 < ε < a). Then the regularized trace formula of problem (1) has the form

$$\sum_{m=0}^{\infty} \varepsilon_m \sum_{n=0}^{\infty} [\lambda_{mn} - (2m + 4n + 2) - A_m/(n + 1)^{1/2} - c_0 A_m/(c\,(n + 1)^{3/2})] = 0,$$

where

$$A_m = 1/\pi \int_{\varepsilon}^{a} (q(r)/r^m)\,dr, \quad c = \sum_{n=0}^{\infty} 1/(n + 1)^{3/2},$$
$$\varepsilon_0 = 1, \ \varepsilon_1 = \varepsilon_2 = \ldots = 2,$$

and the constant c_0 is determined by Eq. (9).

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AN INDICATOR OF THE NONCOMPACTNESS OF A FOLIATION ON M_g^2

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1. Preliminary Definitions. Let us consider a closed form ω defined on a manifold M and possessing nondegenerate isolated singularities.

Definition 1 [1]. A point $p \in M$ is called a regular singularity of ω, if in some neighborhood O(p) ω = df, where f is a Morse function, having a singularity at p.

The form ω determines a foliation F_ω on the set M − Sing ω.

Definition 2. Let us consider γ-nonsingular compact leaves F_ω and the mapping $\gamma \to [\gamma] \in H_1(M_g^2)$. Its image generates a subgroup in $H_1(M_g^2)$. Let us denote it by H_ω.

Definition 3. [2] Let $[z_1], \ldots, [z_{2g}]$ be some basis of cycles in $H_1(M_g^2)$, then

$$\operatorname{dirr} \omega = \operatorname{rk}_{\mathbf{Q}} \left\{ \int_{z_1} \omega, \ldots, \int_{z_{2g}} \omega \right\} - 1.$$

By M_ω let us denote the set obtained by discarding all maximal neibhborhoods consisting of diffeomorphic compact leaves and all leaves which can be compactified by adding singular points.

It was proved in [2] that in the case dirr ω ≤ 0 always $M_\omega = \varnothing$. The object of this paper is to indicate for a foliation on the surface M_g^2 a sufficient condition that $M_\omega \neq \varnothing$. Namely (Theorem 2): if g ≠ 0 and dirr ω ≥ g, then $M_\omega \neq \varnothing$.

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2. Fundamental Theorem. At first let us prove the following criterion.

THEOREM 1. $M_\omega = \varnothing \Leftrightarrow \operatorname{rk} H_\omega = g$.

Proof. 1. Let $M_\omega = \varnothing$, then F_ω contains two types of leaves:

1) Leaves which can be compactified by adding to them singular points. Let us denote these leaves γ_k^0. There is a finite number of them, since the singularities of the form ω are isolated.

2) Nonsingular compact leaves.

Let γ be a nonsingular compact leaf ω. By $O(\gamma)$ let us denote a maximal neighborhood consisting of leaves diffeomorphic to it. It is a cylinder with generator γ. The boundary $\overline{O}(\gamma)$ contains at least one critical point of the form ω. In view of the nondegeneracy of the singularities each critical point can be contained in the boundary of no more than four cylinders. Thus, the number of cylinders does not exceed 4 × number of critical points. Consequently, M_g^2 is representable in the form

$$M_g^2 = \cup_{n=1}^N O(\gamma_n) \cup_{k=1}^K \gamma_k^0 \cup_{i=1}^I p_i, \tag{1}$$

where p_i are the singular points of the form ω, N, k, $I < \infty$.

2. Let us consider the (generated by the foliation F_ω) group H_ω.

LEMMA. $\operatorname{rk} H_\omega = 0 \Leftrightarrow g = 0$.

Proof. In one direction the assertion is trivial. Now let $\operatorname{rk} H_\omega = 0$. We represent M_g^2 in the form (1). Let us denote $V_n = \overline{O}(\gamma_n)$ and $T = \cup_{n=1}^N \partial V_n$. The boundary of the cylinder V_n is homologous to a generator. Then $M_g^2 = \cup_{n=1}^N V_n$, where $V_i \cap V_j \subset T$. Let us consider an arbitrary cycle $[z] \in H_1(M_g^2)$. By hypothesis $\operatorname{rk} H_\omega = 0$; consequently, $\gamma_n \sim 0$, $n = 1, \ldots, N$. Then $z \circ \gamma_n = 0$ and $z \circ T = 0$. There exists $z' \in [z]$ such that $z' \cap T = \varnothing$. Thus, $z' = \cap m_i \gamma_i$, $\gamma_i \in V_i$. Since $\gamma_i \sim 0$, then also $z' \sim 0$. Consequently, $[z] = 0$. The lemma is proved.

3. If $\operatorname{rk} H_\omega = 0$, then everything is proved. Let us consider the case $\operatorname{rk} H_\omega = k \neq 0$.

Let $k = 1$ and $[\gamma]$ be a generator of H_ω. Let us section M_g^2 according to the corresponding leaf γ. Since M_g^2 is orientable, then in the presence of the section a surface is obtained, the boundary of which consists of two closed curves, each of which is a leaf. To each of the two being formed boundaries we attach the circle D^2. There is obtained a closed surface M':

$$\chi(M') = \chi(M_g^2) + 2\chi(D^2) = 2 - 2(g-1).$$

Due to the classification theorem on surfaces $M' = M_{g-1}^2$.

Let $k > 1$ and $H_\omega = \langle[\gamma_1], \ldots, [\gamma_k]\rangle$. By $\Gamma = \{\gamma_1, \ldots, \gamma_k\}$ let us denote the set of cut leaves. In a similar manner, after the cutting of M_g^2 according to Γ and the attaching of 2k disks we obtain $M' = M_{g-k}^2$.

Let us retract each of the attached disks to a point, let us denote these points d_i, $i = 1, \ldots, 2k$. On $M' \setminus \cup_{i=1}^{2k} d_i$ there is defined a form ω' such that $\omega'|_{M' \setminus \cup d_i} = \omega|_{M \setminus \Gamma}$. It is closed and determines on the set $M' \setminus \cup_{i=1}^{2k} d_i \setminus \operatorname{Sing} \omega$ the foliation F_ω'. Let us consider H_ω'. Let $[\gamma] \in H_{\omega'}$. Then $[\gamma] \in H_\omega$ and $\gamma - \sum m_i \gamma_i = \partial V$ and V is a sheet. Let V' be the image of V in M'. It is evident that $\partial V' = \gamma$. Consequently, $H_\omega' = 0$. Then by the lemma $M_{g-k}^2 = S^2$ and $k = g$. The theorem is proved in one direction.

4. Now let $\operatorname{rk} H_\omega = g$. Then $H_\omega = \langle[\gamma_1], \ldots, [\gamma_g]\rangle$, where $\gamma_i \in F_\omega$. Let us cut M_g^2 according to these leaves. Similarly to how this was done in Sec. 3, we obtain $M' = S^2$ and on it the foliation F_ω'. Since $d\omega' = 0$ and $H^1(S^2) = 0$, then the form is exact and the foliation F_ω' is compact. Consequently, the foliation F_ω is also compact. Theorem 1 is proved.

THEOREM 2. Let ω be a closed form with More singularities, given on M_g^2, $g \neq 0$ such that $\operatorname{dirr} \omega \geq g$. Then the foliation F_ω has a noncompact fiber.

Proof. Let us assume the contrary: $M_\omega = \varnothing$. Then by Theorem 1: $\operatorname{rk} H_\omega = g$. Let $\gamma_1, \ldots, \gamma_g \in F_\omega$ be such that $H_\omega = \langle[\gamma_1], \ldots, [\gamma_g]\rangle$. Let us complement H_ω up to a basis in $H_1(M_g^2)$: $[z_1], \ldots, [z_g]$. Then

$$\operatorname{dirr} \omega = \operatorname{rk}_{\mathbb{Q}} \left\{ \int_{z_1} \omega, \ldots, \int_{z_g} \omega \right\} - 1 \leq g - 1.$$

A contradiction. Theorem 2 is proved.

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